Sec. 5 Skill Refresher: Logarithms and Their Properties

Properties of Logarithms:

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $a^{\log_a M} = M$
4. $\log_a a^r = r$
5. $\log_a (MN) = \log_a M + \log_a N$
6. $\log_a (\frac{M}{N}) = \log_a M - \log_a N$
7. $\log_a M^r = r \log_a M$
8. If M = N, then $\log_a M = \log_a N$.
9. If $\log_a M = \log_a N$, then M = N.
10. $\log_a M = \dfrac{\log_b M}{\log_b a}$
11. $\log_a M = \dfrac{\log M}{\log a}$
12. $\log_a M = \dfrac{\ln M}{\ln a}$

Ex. Write $\log_a (x\sqrt{x^2+1})$ as a sum of logarithms and express all powers as factors.

$\log_a x + \log_a (x^2+1)^{\frac{1}{2}}$

$\log_a x + \frac{1}{2}\log_a (x^2+1)$

Ex. Write $\ln \dfrac{x^2}{(x-1)^3}$ as a difference of logarithms and express all powers as factors.

$\ln x^2 - \ln (x-1)^3$

$2 \ln x - 3 \ln (x-1)$

Ex. Write $\log_a \dfrac{x^3\sqrt{x^2+1}}{(x+1)^4}$ as a sum and difference of logarithms and express all powers as factors.

$\log_a x^3 + \log_a (x^2+1)^{\frac{1}{2}} - \log (x+1)^4$

$3 \log_a x + \frac{1}{2} \log_a (x^2+1) - 4 \log (x+1)$

Ex. Simplify and evaluate when possible.

a. $\log_a 7 + 4\log_a 3$

$\log a\, 7 + \log a\, 3^4$

$\log a\,(7 \cdot 3^4)$

$\log a\,(7 \cdot 81)$

$\log a\,(567)$

b. $\log_a x + \log_a 9 + \log_a (x^2+1) - \log_a 5$

$\log a\left(\frac{x \cdot 9(x^2+1)}{5}\right)$

$\log a\left(\frac{9x}{5}(x^2+1)\right)$

$\sqrt[3]{8} = 2$

$2^2 = 4$

c. $\frac{2}{3}\ln 8 - \ln(3^4 - 8)$

$\ln\left(8^{\frac{2}{3}}\right) - \ln(81-8)$

$\ln 4 - \ln 73$

$\ln\left(\frac{4}{73}\right)$

-2.904

d. $\log_5 89$

$\frac{\log_{10} 89}{\log_{10} 5}$

2.789

e. $\log_{\sqrt{2}} \sqrt{5}$

$\frac{\log_{10}\sqrt{5}}{\log_{10}\sqrt{2}}$

2.322

f. $\log_2 6 * \log_6 4$

$\frac{\cancel{\log 6}}{\log 2} \cdot \frac{\log 4}{\cancel{\log 6}}$

$\frac{\log 4}{\log 2}$

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Solving Logarithmic Equations:

Ex. Solve $2\log_5 x = \log_5 9$.

$\log_5 x^2 = \log_5 9$

$x^2 = 9$

$\boxed{x = \pm 3}$

Ex. Solve $\log_4(x+3) + \log_4(2-x) = 1$.

$\log_4(x+3) + \log_4(2-x) = \log_4 4$

$\log_4((x+3)(2-x)) = \log_4 4$

$(x+3)(2-x) = 4$

$2x - x^2 + 6 - 3x = 4$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x+2=0 \qquad x-1=0$

$\boxed{x=2} \qquad \boxed{x=1}$

Solving Exponential Equations:

Ex. Solve $2^x = 5$.

$\log 2^x = \log 5$

$\dfrac{x \cdot \log 2}{\log 2} = \dfrac{\log 5}{\log 2}$

$\boxed{x = 2.322}$

Ex. Solve $8 * 3^x = 5$.

$\dfrac{8 * 3^x}{8} = \dfrac{5}{8}$

$3^x = \frac{5}{8}$

$\log 3^x = \log\left(\frac{5}{8}\right)$

$\dfrac{x \cdot \log 3}{\log 3} = \dfrac{\log\left(\frac{5}{8}\right)}{\log 3}$

$\boxed{x = -.428}$

Ex. Solve $5^{x-2} = 3^{3x+2}$.

$\log(5^{x-2}) = \log(3^{3x+2})$

$(x-2)\log 5 = (3x+2)\log 3$

$x \cdot \log 5 - 2\log 5 = 3x \cdot \log 3 + 2\log 3$

$x\log 5 - 3x\log 3 = 2\log 3 + 2\log 5$

$\dfrac{x(\log 5 - 3\log 3)}{(\log 5 - 3\log 3)} = \dfrac{2\log 3 + 2\log 5}{(\log 5 - 3\log 3)}$

$\boxed{x = -3.212}$

Ex. Solve $4^x - 2^x - 12 = 0$.

$(2^2)^x - 2^x - 12 = 0$

$2^{2x} - 2^x - 12 = 0$

$(2^x)^2 - 2^x - 12 = 0$

$(2^x - 4)(2^x + 3) = 0$

$2^x - 4 = 0 \qquad 2^x + 3 = 0$

$2^x = 4 \qquad 2^x = 3$

$\boxed{x = 2} \qquad \log 2^x = \log 3$

$\dfrac{x \cdot \log 2}{\log 2} = \dfrac{\log 3}{\log 2}$

$\boxed{x = 1.585}$

HW: pg 222, #3-48 (m/3)